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Review Article

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Nonstationary Frequency Conversion of Ultrashort Radiation Pulses

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ABSTRACT
The process of nonlinear interaction of a ultrafast pulse with a longer pulse of high intensity idler wave is considered. Using the Fourier transformations the set of truncated equations is solved to obtain expression for the complex amplitude of ultrashort pulse of radiation. An effect of dispersive features of the medium on the spectral density of a pulse is analyzed. Analytical expressions for the amplitude, spectral density of signal wave are obtained in the constant field approximation. Dependences of spectral density at various values of characteris-tic lengths were revealed. It was found that at larger nonlinear lengths or when characteristic lengths of group velocity mismatch and group velocity dispersion are less than the nonlinear length of medium the excited pulse is separated into several spikes.

Keywords: Metamaterial, Laser Pulse Spectrum, Dispersion Theory, Group Velocity Mismatch, Group Velocity Dispersion.

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Introduction

Amplification of weak signals and broadening the frequency range of laser radiation requires the depelopment of optical parametrical amplifiers. An interest to the nonstationary interaction of ultra pulses is related to the creation of sources of pulses for which the durations are measured in femtoseconds. First theoretical studies of such parametrical amplifiers were performed by [1] The character of interaction between ultrashort modulated pulses depends mainly on the dipspersive properties of medium [2].

Metamaterials with negative refractive index are attractive with a specific features of interaction with electromagnetic waves. The frequency conversion in those materials were mainly investigated by us in the constant intensity approximation [3-6]. This dependence seems more obviously with decrease in the pulse duration. In metamaterials, unlike group ve-locities the phase velocities are in the same direction. However, in dispersive media, the dif-ference in the rates of change of the frequency components of the pulse causes non-stationary distortion of the pulse. Such distortion is especially pronounced for ultrashort pulses and is of great importance for pulses with durations in the femtosecond range. Therefore, when study-ing the nonlinear parametric interaction of ultrashort pulses in metamaterials both phase veloci-ty dispersion and group velocity dispersion must be taken into account simultaneously.

Theory

In this work the nonlinear optical processes are analyzed by the set of truncated equations presented in the nonstationary form. Those

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equations include both the first and second order partial derivatives with respect to time for the complex amplitude of pulse. Analytical expressions for the pulse spectral density and energy were obtained by taking into account the effects of group velocity difference as well as the group velocity dispersion via Fourier description.

Taking into account negative values of dielectric permittivity and magnetic permeability at the frquency of signal wave ω_1, and positive values at frequencies of pump [1]:

$$\left(\frac{\partial}{\partial z}+\frac{1}{u_1}\frac{\partial}{\partial t}-i\frac{g_1}{2}\frac{\partial^2}{\partial t^2}+\delta_1\right)A_1=-i\gamma_1 A_3 A_2^* e^{i\Delta z},$$
$$\left(\frac{\partial}{\partial z}+\frac{1}{u_2}\frac{\partial}{\partial t}-i\frac{g_2}{2}\frac{\partial^2}{\partial t^2}+\delta_2\right)A_2=-i\gamma_2 A_3 A_1^* e^{i\Delta z},\qquad (1)$$
$$\left(\frac{\partial}{\partial z}+\frac{1}{u_3}\frac{\partial}{\partial t}-i\frac{g_3}{2}\frac{\partial^2}{\partial t^2}+\delta_3\right)A_3=-i\gamma_3 A_1 A_2 e^{-i\Delta z},$$

where A_j - refer to the complex amplitudes of interacting waves at frequencies ω_j δ_j - indicate absorption coefficients at those frequencies, $\Delta= k_1- k_2 - k_3$ is the phase mismatch between the interacting waves, gj = $(\partial^2 k_j)/(\partial\omega_j^2)$ (the 3-rd term in the Taylor expansion around the central frequency ω_0: $\Delta\omega = \omega - \omega_0$, $k_n(\omega) \cong k_n(\omega_0) + k_n'\Delta\omega + 1/2\, k_n''\,\Delta\omega^2 + ...$) is the dispersion of group velocities u_j –are the group velocities of waves at frequencies ωj (j=1-3), and γ_j – are the coefficients of nonlinear coupling:

$$\gamma_1=\frac{8\pi\chi_{eff}^2\omega_1^2|\varepsilon_1|}{k_1c^2},\gamma_2=\frac{8\pi\chi_{eff}^2\omega_2^2\varepsilon_2}{k_2c^2},\gamma_3=\frac{8\pi\chi_{eff}^2\omega_3^2\varepsilon_3}{k_3c^2}.$$

Here χ_{eff}^2 - is the effective susceptibility of medium.

Difference in group velocities is given by $\nu=1/u_2 - 1/u_1$ and to solve above set the new variable $\eta=t-1/u_1$ is introduced. If the field at the frequency ω3 is strong and remains un-depleted in the process of nonlinear optical interaction $A_3(z,t)=A_{30}$. If we take into account that during the interactin of waves the pump wave with frequency ω_2 has a large intensity or its amplitude remains constant ($A_2=A_{20}=const,(dA_2/dz)=0$), then the system of equations (1) describing the interaction of the waves with local time variable can be written as follows:

$$\left(\frac{\partial}{\partial z}-\delta_1\right)A_1(z,\eta)=i\gamma_1A_3A_2^*(z,\eta)e^{i\Delta z},$$
$$\left(\frac{\partial}{\partial z}-\nu\frac{\partial}{\partial\eta}+\delta_3\right)A_3(z,\eta)=-i\gamma_3A_1A_{20}(z,\eta)e^{-i\Delta z}.\qquad (2)$$

where $\nu=1/u_2+1/|u_1|$ is the difference in inverse values of group velocities.

Using for amplitudes $A_1(z,\ \eta)$ and $A_3(z,\ \eta)$ the Fourier transformations

$$A_{1,3}(z,\eta)=\int_{-\infty}^{+\infty}A_{1,3}(z,\omega)e^{-i\omega\eta}d\omega,$$

leads to the folowing set

$$\left(\frac{\partial}{\partial z}-\delta_1\right)A_1(z,\omega)=+i\gamma_1A_3A_{20}^*(z,\omega)e^{i\Delta z},$$
$$\left(\frac{\partial}{\partial z}+i\nu\omega+\delta_3\right)A_3(z,\omega)=-i\gamma_3A_1A_{20}(z,\omega)e^{-i\Delta z}.\qquad (3)$$

Differentiation with respect to the z- coordinate gives:

$$\frac{\partial^2a_1(z,\omega)}{\partial z^2}-\zeta_1\frac{\partial a_1(z,\omega)}{\partial z}-\gamma_1\gamma_3I_{20}a_1(z,\omega)=0.\qquad (4)$$

where $a_1(z,\omega)=A_1(z,\omega)e^{-\delta_1 z}$ and $\zeta_1=-\delta_1-\delta_3+i(\Delta-\nu\omega)$ substitutions were made.

By applying to the equiation the bondary conditions
$A_1\ (z=l)=0, A_{2,3}\ (z=0)=A_{20,30}$ (5)

For the complex amplitude yields:

$$A_1(\omega,z)=\frac{i\gamma_1A_{30}A_{20}}{\lambda-k\tan\lambda l}(\sin\lambda z-\tan\lambda l\cdot\cos\lambda z)e^{kz},\qquad (6)$$

where $\lambda=(a+ib)^{1/2}$, $a = -I_2^2 - (\Delta - \nu\omega)^2/4-\delta^2$, $b=(\Delta - \nu\omega)\delta$, $\delta = (\delta_1+\delta_3)/2$, $I_2^2=\gamma_1\gamma_3I_{20}$, $k=(\delta_1-\delta_3)/2+i((\Delta - \nu\omega)/2)$.

Assume, an idler wave has a Gauss profile at the input of metamaterial

$$A_{30}(t)=A_{30}e^{-\frac{t^2}{2\tau^2}+iG\frac{t^2}{2}}.\qquad (7)$$

Here τ –is the duration of an idler pulse, G – refers to the parameter of phase modulation.

The Fourier description of (7) is presented by:

$$A_{30}(\omega)=\frac{A_{30}}{2\pi}\int_{-\infty}^{+\infty}e^{-\frac{t^2}{2\tau^2}+iG\frac{t^2}{2}}e^{+i\omega t}dt.\qquad (8)$$

Hence

$$A_{30}(\omega)=\frac{A_{30}}{\sqrt{2\pi}}\frac{\tau}{\sqrt{1+iC\tau^2}}\cdot\exp\left(-\frac{\omega^2\tau^2}{2(1+iG\tau^2)}\right).\qquad (9)$$

A spectral density $S_{30}(\omega,z) = A_{30}(\omega,z)\cdot A_{30}^*(\omega, z)$ of an idler wave then is given:

$$S_{30}(\omega)=\frac{cn_3A_{30}^2}{16\pi^2}\frac{\tau^2}{\sqrt{1+C^2\tau^4}}\cdot exp\left(-\frac{\omega^2\tau^2}{1+G^2\tau^4}\right).\qquad (10)$$

Taking into account Eq.-s (6-10) for the spectral density of a signal wave we obtain:

$$S_1(\omega,z)=D\frac{(\sin\lambda'z-tg\lambda'l\cdot\cos\lambda'z)^2}{\lambda'^2+\frac{k^2}{4}\tan^2\lambda'l}\cdot\exp\left(-\frac{\omega^2\tau^2}{1+G^2\tau^4}\right).\qquad (11)$$

where $D= cn_1\gamma_1^2 I_{30} I_{20}\,\tau^2/16\pi^2$, *and* $k=\omega^2 g/2 - \omega\nu - \Delta$.

Since the describing of the shape of signal wave spectrum is reasonable through the ratios $l_{n/l}/l_d$ and $l_{n/l}/l_v$ the last equation encloses the expressions for λ' and k:

$$\lambda'=l_{n/l}{}^{-1}\left[\frac{1}{4}\left(\frac{1}{2}(\alpha-1)\frac{l_{n/l}}{l_{d.}}\omega^2\tau^2+\frac{l_{n/l}}{l_v}\omega\nu-\frac{\Delta}{\Gamma_3}\right)^2-1\right]^{1/2},$$
$$k=l_{n/l}{}^{-1}\left[i\left(\frac{1}{4}(\alpha-1)\frac{l_{n/l}}{l_{d.}}\omega^2\tau^2+\frac{l_{n/l}}{l_v}\omega\nu-\frac{\Delta}{\Gamma_3}\right)\right],\qquad \alpha=\frac{g_2}{g_1}.$$

Results and Discussion

Analysis has showed that with increase of ratio $l_{n/l}/l_v$ the width of maxima of spectral densities decreases and the maxima displace toward lower values of $\omega\tau$. It was obtained that maxima of spectral density decrease with increase of ratio $z/l_{n/l}$. Such a decrease also is observed when the phase modulation parameter increases. Under phase matching conditions dependences of spectral density are symmetric relatively positive and negative values of $\omega\tau$ parameter.

The shape of amplifying signal pulse is determined not by the quantities z, $l_{n/l}$ and l_v but with the ratios $z/l_{n/l}$ və $l_{n/l}/l_v$. Dependences of spectral density of a signal pulse on the frequency modulation parameter $\omega\tau$ at different the ratio $l_{n/l}/l_v$ are presented in Figure.1.

Ass can be seen the shpae of a signal pulse chages with variation of the ratio $l_{n/l}/l_v$. Up to definite values of this ratio the width of a spectrum becomes narrower (curves 1-6) the maxima of those curves displace towsard smaller values of frequency modulation parameter ωτ. At larger values suc a displacement is not observed. (curves 1 and 2).

Figure.2 demonstrates dependence of signal pulse density on the parameter ωτ at different values of ratios $l_{n/l}/l_v$ və $l_{n/l}/l_d$.

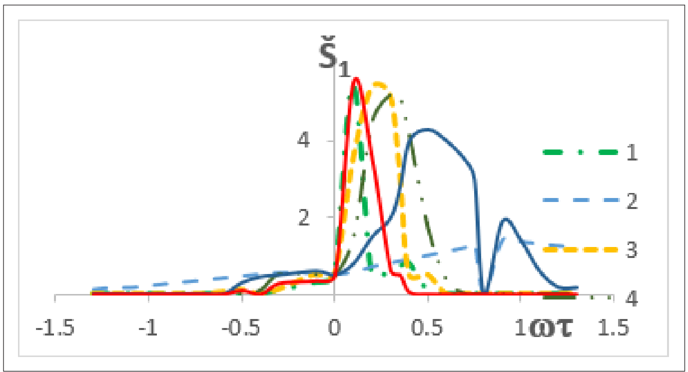

Figure 1: Spectral density of a signal pulse as a function of frequency modulation parameter ωτ at different values of the ratio $l_{n/l}/l_v$: *($p=0$, $z/l_{q/l}=0{,}5$, $\Delta l_{n/l}/2=1{,}\delta_i=0$) $l_{n/l}/l_v$: 1-$l_{n/l}/l_v=20$; 2-$l_{n/l}/l_v=15$; 3-$l_{n/l}/l_v=10$; 4-$l_{n/l}/l_v=6$; 5-$l_{n/l}/l_v=3$; 6-$l_{n/l}/l_v=1$.*

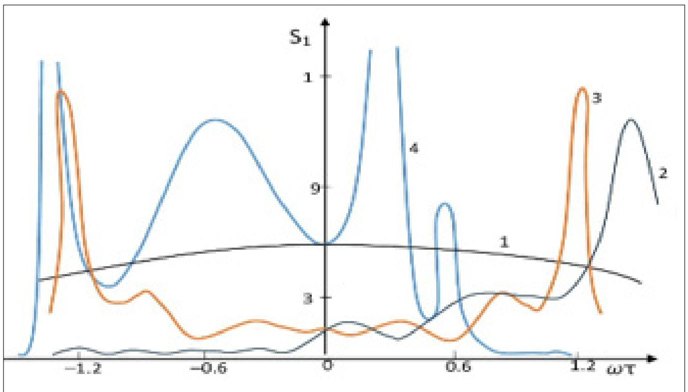

Figure 2: Dependence of a signal pulse density $S_1(\omega,z)$ on the frequency modulation parameter $\omega\tau$: *($\Delta=0$, $\delta_i=0$,$p=5$, $z/l_{n/l}=0{,}5$: 1-$l_{n/l}/l_v=0$; 2-$l_{n/l}/l_v=3$, $l_{n/l}/l_d=0$; 3-$l_{n/l}/l_v=0$,$l_{n/l}/l_d=3$; 4-$l_{n/l}/l_v=3$,$l_{n/l}/l_d=3$.*

It is seen that variation in these ratios leads to the change in the spectral density of a signal pulse. When the condition of və $ln/l/l_v=0$ is fullfiled dependences become symmetrical relatively positive and negative values of frequency modulation parameter (curves 1 and 3). Note, that these dependences are obtained for the case when signs of dispersion coefficients of group velocities. If $g_1=g_2$ the signal pulse is amplified without dispersion of group velocities. The plots are obtained for the case of $g_2/g_1=3$.

Conclusion

On the basis on the obtained results we can conclude that when the nonlinear length of the medium is more less than the kuasistatic length dependence of spectral density on the frequency modulation parameter is symmetric relatively values of different signs. Up to definite values of this ratio the width of a spectrum becomes narrower and the maxima of those curves displace toward smaller values of frequency modulation parameter ωτ.

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