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Correcting a 300 Year Surviving Error: The Correct Expression of the Euler Acceleration

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ABSTRACT
In the current paper we correct an error that has survived for almost 300 years: the expression of the Euler acceleration. The solution is of great interest for real time applications because earth-bound laboratories are inertial only in approximation. The accelerations play an important role in centrifuges ramping up to speed. We are correcting not only the expression of the Euler force in its classical (non-relativistic) form by correcting an error in its derivation that has persisted for nearly 300 years but also the general expression of the Coriolis acceleration for the case of accelerating rotation.

Keywords: General Coordinate Transformations, Non-uniform Rotation, Euler Acceleration, Coriolis Acceleration.

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The Case of Non-Uniform Angular Speed, the Emergence of the Euler force

It has been known for over 300 years that in the frame of a uniformly rotating platform fictitious forces like the centrifugal and Coriolis force arise [1, 2]. What happens if the angular speed of the platform ω varies in time? This gives rise to an additional fictitious force, the so-called Euler force [3, 4]. Of course, the centrifugal and the Coriolis force still exist and, in the case of the Coriolis force, they take a more complicated form. All this has been known for a long time. What has not been known is the fact that the derivation of the Euler force (and of the Coriolis force, for the case of time variant angular speed} have been incorrect.

We can trace the error to the use of the fact that the time derivatives of the unit vectors in a **uniformly** rotating frame can be expressed as:

$$\frac{d\mathbf{u}}{dt} = \boldsymbol{\omega} \times \mathbf{u} \tag{1.1}$$

But (1.1) is true only in the case of constant angular speed, ω it is obviously not true for variable angular speed. Indeed, let's consider, for example the unit vector of the rotating x-axis, ux:

$$\mathbf{u_x} = (\cos\omega t, -\sin\omega t) \tag{1.2}$$

Then:

$$\frac{d\mathbf{u_x}}{dt} = (\omega + t\frac{d\omega}{dt})(-\sin\omega t, -\cos\omega t) \tag{1.3}$$

In the next section we will be providing a rigorous formalism that addresses all the derivations of fictitious accelerations and forces and produces the complete, exact expressions.

Corrected Formalism

In this section we will present a matrix-based approach to deriving the compact, exact expressions for both fictitious acceleration and fictitious forces in the relativistic sector. We start with:

$$\begin{aligned}
&\mathbf{r} = R\mathbf{r}' \\
&\mathbf{r} = \begin{bmatrix} x \\ y \end{bmatrix} \\
&\mathbf{r}' = \begin{bmatrix} x' \\ y' \end{bmatrix} \\
&R = \begin{bmatrix} \cos\omega t' & -\sin\omega t' \\ \sin\omega t' & \cos\omega t' \end{bmatrix}
\end{aligned} \tag{2.1}$$

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Then:

$$\dot{\mathbf{r}} = \frac{d\mathbf{r}}{dt} = \dot{R}\mathbf{r}' + R\dot{\mathbf{r}}' = \dot{R}R^{-1}\mathbf{r} + R\dot{\mathbf{r}}' = \dot{R}R^{T}\mathbf{r} + R\dot{\mathbf{r}}' \tag{2.2}$$

The over dot represents differentiation with respect to the coordinate time, t and $R^{-1} = R^{T}$. It follows immediately that:

$$\dot{\mathbf{r}}' = R^{T}(\dot{\mathbf{r}} - \dot{R}R^{T}\mathbf{r}) \tag{2.3}$$

After some elementary algebraic manipulation, we get:

$$\dot{\mathbf{r}}' = R^{T}\dot{\mathbf{r}} - (\omega + t\dot{\omega})Q\mathbf{r}$$
$$Q = \begin{bmatrix} \sin\omega t & \cos\omega t \\ -\cos\omega t & \sin\omega t \end{bmatrix} \tag{2.4}$$

The resulting fictitious acceleration in the rotating frame is:

$$\begin{aligned} \ddot{\mathbf{r}}' &= \dot{R}^{T}\dot{\mathbf{r}} + R^{T}\ddot{\mathbf{r}} - (2\dot{\omega} + t\ddot{\omega})Q\mathbf{r} - (\omega + t\dot{\omega})\dot{Q}\mathbf{r} - (\omega + t\dot{\omega})Q\dot{\mathbf{r}} = \\ &= \dot{R}^{T}\dot{\mathbf{r}} - (2\dot{\omega} + t\ddot{\omega})Q\mathbf{r} - (\omega + t\dot{\omega})\dot{Q}\mathbf{r} - (\omega + t\dot{\omega})Q\dot{\mathbf{r}} \end{aligned} \tag{2.5}$$

After some more manipulation, noting that

$$\begin{aligned} \dot{Q} &= (\omega + t\dot{\omega})R^{T} \\ \dot{R}^{T} &= -(\omega + t\dot{\omega})Q \end{aligned} \tag{2.6}$$

we obtain the final expression for the classical fictitious acceleration:

$$\ddot{\mathbf{r}}' = -2(\omega + t\dot{\omega})Q\mathbf{v} - [(\omega + t\dot{\omega})^{2}R^{T} + (2\dot{\omega} + t\ddot{\omega})Q]\mathbf{r} \tag{2.7}$$

One last step is to re-order (2.7) after the powers of ω and its derivatives:

$$\ddot{\mathbf{r}}' = -\omega^{2}R^{T}\mathbf{r} - 2(\omega + t\dot{\omega})Q\mathbf{v} - [(2\dot{\omega} + t\ddot{\omega})Q + (t\dot{\omega})^{2}R^{T} + 2t\omega\dot{\omega}R^{T}]\mathbf{r} \tag{2.8}$$

Separating the components, we obtain:

$$\begin{aligned} \mathbf{a}'_{centrifugal} &= -\omega^{2}R^{T}\mathbf{r} \\ \mathbf{a}'_{coriolis} &= -2(\omega + t\dot{\omega})Q\mathbf{v} \\ \mathbf{a}'_{euler} &= -[(2\dot{\omega} + t\ddot{\omega})Q + (t\dot{\omega})^{2}R^{T} + 2t\omega\dot{\omega}R^{T}]\mathbf{r} \end{aligned} \tag{2.9}$$

It is interesting to note that not only the Euler acceleration looks different but so does the Coriolis acceleration, all due to the fact that the angular speed is variable. A quick sanity check recovers the classical case for ω =constant:

$$\begin{aligned} \mathbf{a}'_{centrifugal} &= -\omega^{2}R^{T}\mathbf{r} \\ \mathbf{a}'_{coriolis} &= -2\omega Q\mathbf{v} \\ \mathbf{a}'_{euler} &= 0 \end{aligned} \tag{2.10}$$

The Euler acceleration in all existing literature since its initial derivation is missing the explicit term in time, $-[(t\ddot{\omega})Q + (t\dot{\omega})^{2}R^{T} + 2t\omega\dot{\omega}R^{T}]\mathbf{r}$

The centrifugal acceleration is the only acceleration not affected by the error outlined in (1.3).

An AI Simulation

Meta AI responded to the new formulas for the accelerated motion with the following simulation of the old derivation versus the new derivation:

```
import numpy as np
import matplotlib.pyplot as plt

# Define time-dependent angular velocity
omega_0 = np.array([0, 0, 1]) # Initial angular velocity (rad/s)
domega_dt = np.array([0, 0, 0.1]) # Angular acceleration (rad/s^2)

def omega(t):
return omega_0 + domega_dt * t # Linearly increasing angular velocity

def euler_force_traditional(r, t, m=1):
return -m * np.cross(domega_dt, r) # Traditional formula

def euler_force_corrected(r, t, m=1):
return -m * (np.cross(domega_dt, r) + np.cross(omega(t), np.cross(omega(t), r)))

# Simulation parameters
time = np.linspace(0, 10, 100) # Simulate for 10 seconds, 100 steps r = np.array([1, 0, 0]) # Position vector in rotating frame

# Compute forces
traditional_forces = np.array([euler_force_traditional(r, t) for t in time]) corrected_forces = np.array([euler_force_corrected(r, t) for t in time])
diff_forces = corrected_forces - traditional_forces # Difference due to correction # Plot results

fig, ax = plt.subplots(3, 1, figsize=(8, 10))
labels = ['X Component', 'Y Component', 'Z Component']

for i in range(3):
ax[i].plot(time, traditional_forces[:, i], label='Traditional', linestyle='--') ax[i].plot(time, corrected_forces[:, i], label='Corrected', linestyle='-') ax[i].plot(time, diff_forces[:, i], label='Difference', linestyle=':') ax[i].set_ylabel(labels[i])
ax[i].legend()

ax[2].set_xlabel('Time (s)')
plt.suptitle('Comparison of Traditional vs Corrected Euler Force') plt.show()
```

Conclusions and Future Work

We corrected an error in the expression of the Euler acceleration/force that has persisted for nearly three hundred years. As a byproduct, we also derived the correct expression of the Coriolis acceleration for the case of time varying angular speed. The corrected formulas are important for all mechanical industrial devices, like centrifuges, for example since both the Euler and the Coriolis force turn out to be larger than predicted by previous treatises on classical mechanics. Future work includes the incorporation of the corrected formulas in textbooks and teaching various AIs that the 300 years old formulas need to be corrected.

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