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Tuning Mode Shapes and Modal Frequencies of Simplified Beam Model Using Quantum-behaved PSO and Adaptive PSO

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ABSTRACT

Numerical models are vital tools in structural analysis, but due to necessary simplifications, such as idealizations and assumptions, they often fail to fully capture real-world structural behavior. To reduce this gap, we integrate experimental data with finite element model updating techniques, enhancing the accuracy and reliability of simulations. This study presents a practical and robust methodology for tuning the mode shapes and natural frequencies of a simplified beam-based structural model using advanced optimization algorithms. A complex structure is approximated by an equivalent beam model, with cross-sections carefully chosen to match the local rigidity of the original system. Joint flexibility, a critical factor in capturing realistic dynamic behavior, is modelled using rotational springs to simulate local compliance at structural connections. To improve the fidelity of the model, we apply optimization algorithms like Quantum-behaved PSO and Adaptive PSO. These algorithms adjust geometric dimensions, elastic properties, and joint stiffnesses to minimize the discrepancy between numerical and experimental modal data.

Each method is evaluated based on convergence speed, accuracy in replicating target dynamics, and resistance to local minima. Among them, hybrid and adaptive approaches—QPSO and APSO—demonstrate good performance in both precision and computational efficiency. This work underscores the value of combining optimization algorithms with model updating to produce high-fidelity structural models. The proposed approach offers a powerful tool for dynamic simulation and structural health monitoring in engineering practice.

Keywords: Quantum-behaved PSO(QPSO), Adaptive PSO (APSO), Finite Element Model Updating.

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Nomenclature

N: Total Number of Natural Frequencies.
f^i: Computed Natural Frequency for Mode I.
f_i^{Target}: Target Natural Frequency for the Experimental Mode I.
MAC(ϕ_i,ϕ_j): Modal Assurance Criterion Value Between Computed Mode Shape for JTH Mode Shape and Target Mode Shape for Mode I.
w_{fi}: Weightage on ith Frequency
w_{mi}: Weightage on ith Modal Assurance Criterion.
v^i_t: Velocity of i^th Particle During t^th Iteration
c_1: Cognitive Coefficient
c_2: Social Coefficient
ω: Inertia Weight
$pbest_i$: Best Position of i^th Particle
gbest: Best Position of Entire Swarm

$x_i^{(t+1)}$: Updated Position of Particle
p_i: Personal Best Position,
mbest: Mean Best Position of the Swarm,
β: Contraction-expansion Coefficient,
u: Random Number in (0,1).

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Introduction

The finite element method (FEM) is among the most commonly applied numerical approaches for determining the modal frequencies and mode shapes of engineering structures, ranging from aerospace components and bridges to mechanical assemblies. Through discretization of the structure into smaller elements, FEM enables the accurate prediction of dynamic characteristics that are essential for vibration analysis, structural health monitoring, and design optimization. However, FEM predictions often differ from results obtained through experimental modal analysis (EMA). These discrepancies are unavoidable in practical engineering applications. They can stem from multiple sources: oversimplified modelling assumptions, uncertainties in material properties, idealized boundary conditions, difficulties in accurately representing nonlinearities, or even errors introduced during the measurement process. Such deviations, if left unaddressed, may lead to unsafe designs or inefficient maintenance decisions, making the alignment between computational predictions and experimental observations a critical concern. To mitigate these differences, the finite element model updating (FEMU) technique has been developed and extensively applied [1]. The FEMU process involves constructing an initial finite element model based on best-available assumptions, then iteratively modifying selected parameters—such as stiffness, mass, damping, or boundary constraints—until the numerical predictions closely match the experimental observations [2,3]. This iterative correction not only improves the accuracy of the model but also enhances the understanding of the true physical behaviour of the structure [2]. As a result, FEMU has found applications in structural damage detection, joint flexibility estimation, and performance tuning of complex systems [4-7]. Given the inherently nonlinear and multi-dimensional nature of FEMU problems, optimization algorithms have become indispensable tools for parameter updating. Various metaheuristic approaches have been explored, including Genetic Algorithms (GA), Crow Search Algorithm, Differential Evolution (DE), Firefly Algorithm, Simulated Annealing (SA), Bayesian Optimization, and Particle Swarm Optimization (PSO) [8-14]. Among these algorithms, PSO has emerged as a particularly attractive choice due to its ease of implementation, relatively fast convergence, and ability to handle complex objective functions without requiring gradient information. PSO has been successfully employed in updating mass and stiffness matrices, refining uncertain boundary conditions, and tuning the dynamic responses of large-scale structural systems. Nevertheless, standard PSO is not without its drawbacks. It is prone to premature convergence—settling into local optima before adequately exploring the search space—and stagnation, particularly in high-dimensional or rugged optimization landscapes. To overcome these challenges, advanced PSO variants have been developed. Quantum-behaved PSO (QPSO), inspired by quantum mechanics, introduces a probabilistic position update mechanism in place of the traditional velocity-based approach [15,16]. This modification enhances global search capability and reduces the risk of entrapment in local minima. Similarly, Adaptive PSO (APSO) dynamically adjusts control parameters such as inertia and cognitive/social learning factors during the search process, maintaining an effective trade-off between exploring new regions of the search space and intensively refining the most promising solutions [17,18]. These improvements have made QPSO and APSO valuable tools in high-accuracy FEMU applications, particularly for structural health monitoring and complex dynamic system calibration.

Methodologies

In ANSYS using PYMAPDL (the Python interface for Mechanical APDL), tuning computed modal frequencies to match desired target frequencies involves several essential steps. The process begins with model creation, which includes defining the geometrical parameters, material properties, and selecting appropriate elements within Mechanical APDL. Once the model is defined, meshing is performed to accurately represent the model geometry and to ensure that the finite element analysis captures the necessary details for accurate results. The model is then solved using the appropriate analysis commands to compute the structure's natural frequencies and mode shapes. To adjust these frequencies toward the desired values, optimization techniques are employed. This involves iteratively adjusting the material properties, geometric features, or boundary conditions to modify the modal frequencies. In this work, Particle Swarm Optimization (PSO) and its quantum-behaved variant (QPSO), Adaptive PSO (APSO) were utilized to find the best combination of parameters that achieve the desired modal frequencies and vibration characteristics, allowing for a refined and precise solution. The goal of the optimization is to reduce the discrepancy between the simulated and target mode shapes and natural frequencies obtained from experimental data. The modal frequency and mode shape tuning objective function uses the Modal Assurance Criterion (MAC) to minimize discrepancies between measured and predicted mode shape [19,20].

Objective Function for Tuning:
Minimize:

$$\mathrm{J} = \sum_{\mathrm{i}=1}^{\mathrm{n}} (\mathrm{w_i} \varepsilon_\mathrm{i} + \mathrm{w_{mi}} \varepsilon_{\mathrm{MACi}}) \tag{1}$$

$$\varepsilon_{MACi} = 1 - MAC(\phi_i, \phi_i) \tag{2}$$

$$\varepsilon_{fi} = (f_i - f_i^{Target})^2 \tag{3}$$

$$MAC(\phi_i, \phi_{j_i}) = \frac{|\phi_i \phi_i|^2}{(\phi_i^T \phi_i)(\phi_i^T \phi_i)} \tag{4}$$

Particle Swarm Optimization (PSO) is a nature-inspired optimization method that models the collective behavior observed in flocks of birds. The algorithm optimizes by iteratively improving solutions based on a fitness function. Each candidate solution, referred to as a particle navigates the search space with a velocity that is continuously updated using both its personal experience and the shared knowledge of the entire swarm.

In classical PSO, n particles are initialized randomly in the solution space. Each particle i is defined by a position vector x_i and a velocity vector v_i. The best position visited by a particle is recorded as $pbest_i$, and the optimal position discovered by the swarm as a whole is represented as.

Classical PSO

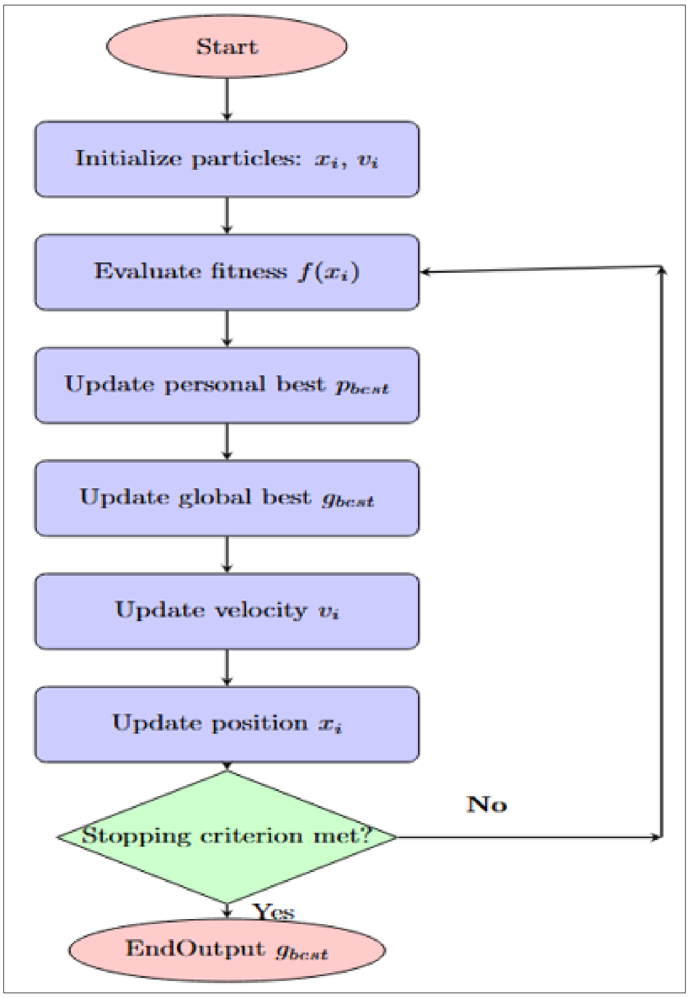

Figure 1: Classical PSO Flowchart

Position and velocity update formula

$$v_i^{(t+1)} = \omega\, v_i^t + r_1 \cdot c_1 \cdot (pbest_i - x_i^t) + r_2 \cdot c_2 \cdot (gbest - x_i^t) \qquad (5)$$

$$x_i^{(t+1)} = x_i^t + v_i^{t+1} \qquad (6)$$

In classical Particle Swarm Optimization (PSO), the inertia weight ω decides how much of a particle's previous speed it keeps—higher values make particles explore more widely, while lower values make them focus on fine-tuning solutions. The cognitive factor c_1 controls how strongly a particle moves toward its own best position found, and the social factor c_2 controls how strongly it moves toward the best position found by the whole group. The random numbers r_1 and r_2 add unpredictability so particles don't all move in the same way and can escape bad spots. Swarm size decides how many particles search at once—more particles explore better but take more time to calculate. The flowchart of classical PSO is shown in Figure 1.

Quantum Behaved PSO

Quantum-behaved Particle Swarm Optimization (QPSO) is an enhanced form of the standard PSO algorithm. It is inspired by the principles of quantum mechanics, particularly the concept that a particle has a probability distribution rather than a deterministic position. Unlike classical PSO, where particles follow Newtonian dynamics, QPSO assumes that particles are governed by a quantum potential well, allowing for a broader and more flexible search in the solution space. In QPSO, each particle has only a position vector x_i, without explicitly maintaining a velocity vector. The position update mechanism is probabilistic and derived from the quantum δ-potential well model. The particle position at iteration t+1 is updated using the equation:

Position update formula

$$x_i^{(t+1)} = p_i \pm \beta \cdot |mbest - x_i^t| \cdot \ln(1/u) \qquad (7)$$

$$mbest = 1/N \sum_{i=}^{N} pbest_i \qquad (8)$$

In Quantum-behaved PSO (QPSO), particles update positions using a quantum model guided by the global best and the swarm's mean best mbest, without using velocity. The key parameter, **contraction–expansion coefficient** β balances exploration (high β) and exploitation (low β) and is often decreased over time. Random numbers add unpredictability to avoid premature convergence, while swarm size controls search coverage and computation. Proper tuning of β and swarm size is essential for fast, effective convergence. The flowchart of Quantum behaved PSO is shown in Figure 2.

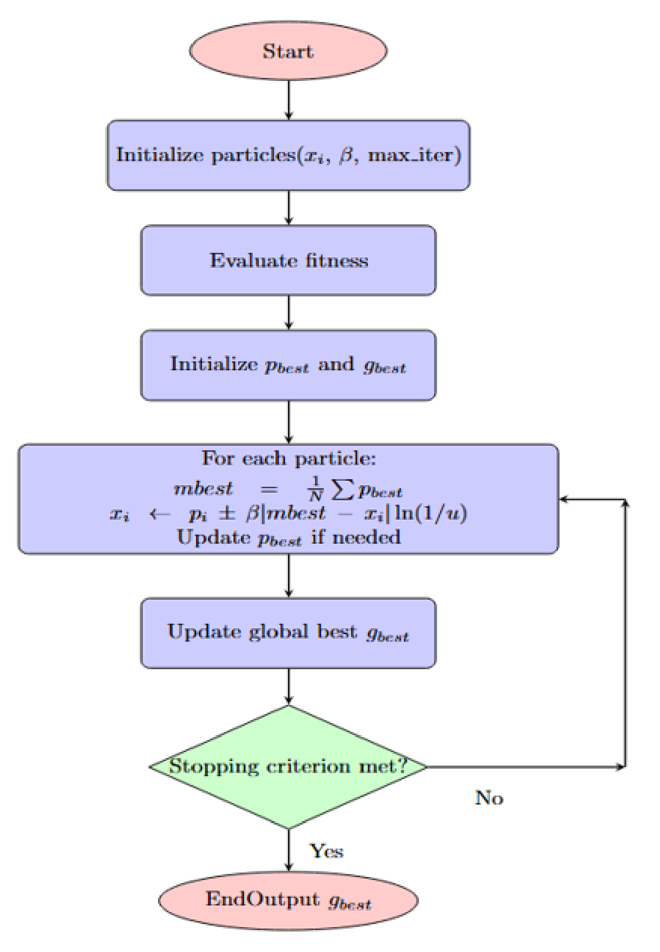

Figure 2: Quantum Behaved PSO Flowchart

Adaptive PSO

Adaptive PSO is an enhanced version of the classical PSO algorithm that dynamically adjusts its control parameters—specifically, the inertia weight and the cognitive and social coefficients—as the optimization progresses. This adaptation allows for improved trade-off between exploration and exploitation, which improves rate of convergence and the optimality of the solution while reducing the risk of premature convergence.

In Adaptive PSO, the velocity and position of each particle are revised using the following expressions:

$$v_i^{(t+1)} = \omega(t)\cdot v_i^t + r_1\cdot c_1(t)\cdot(pbest_i - x_i^t) + r_2 \cdot c_2(t) \cdot (gbest - x_i^t) \quad (9)$$

$$x_i^{t+1} = x_i^t + v_i^{t+1} \quad (10)$$

Here cognitive coefficient, the social coefficient, and the inertia vary with iteration

$$\omega(t) = \omega_{max} - \frac{\omega_{max} - \omega_{min}}{t_{max}} \cdot t \quad (11)$$

$$c_1^t = (c_{1f} - c_{1i}) \cdot \frac{t}{t_{max}} + c_{1i} \quad (12)$$

$$c_2^t = (c_{2f} - c_{2i}) \cdot \frac{t}{t_{max}} + c_{2i} \quad (13)$$

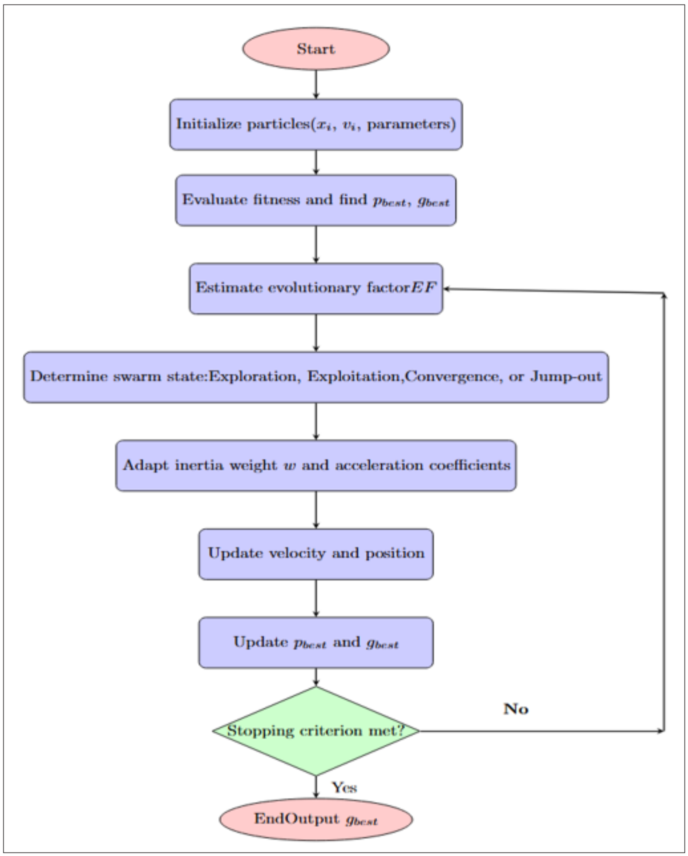

Figure 3: Adaptive PSO Flowchart

This linearly decreasing inertia weight helps transition the algorithm from exploration to exploitation as the iteration count increases. The cognitive coefficient $c_1(t)$ typically decreases over time, reducing individual learning and allowing for more social learning as convergence nears. Conversely, the social coefficient $c_2(t)$ often increases over time, encouraging convergence towards the global best. The flowchart of Adaptive PSO is shown in Figure 3.

Results and Discussion

Problem Setup

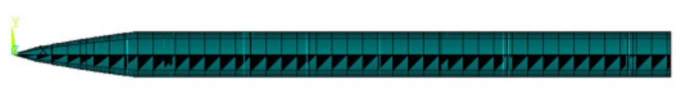

Figure 4: Finite Element Model of the Simplified Beam Model

In this study, the finite element (FE) model of a beam-type structure is updated using the reduced super beam method. This method simplifies complex structures by reducing the number of degrees of freedom while still keeping the important dynamic behaviour accurate. The structure is made up of beam segments connected by bolted joints. The flexibility of each joint is modelled using rotational springs represented by COMBIN14 elements. These elements are considered linear, massless, and of constant stiffness, and they act only in the rotational direction related to the bending plane. Nonlinear effects such as bolt slip, friction, and clearance are ignored.

The beam parts are modelled using BEAM188 elements based on Timoshenko beam theory, assuming the material is linear elastic, isotropic, and homogeneous. Small deformation and vibration are considered, and both shear deformation and rotary inertia are included to better capture the lower bending and torsional modes. The structure is analyzed under free–free boundary conditions, similar to experimental suspension, so that the natural frequencies and mode shapes are not affected by supports or constraints.

The model is divided into substructures. For each substructure, the thickness, Young's modulus, and rotational stiffness of the connecting springs are used as updating variables within practical limits. Finally, the beam model is updated and matched with the target modal frequencies and mode shapes obtained from experimental modal analysis. The target and initial modal frequencies are listed in Table 1.

Table 1: Target Modal Frequencies

Mode	Target Frequency	Initial Frequency	Initial Error (%)	Initial MAC Error ((1-MAC)×100)
1st Bending Mode	31.73	33.3987085	5.2590	7.0568
2nd Bending Mode	73.96	70.5289132	4.6391	6.7312
3rd Bending Mode	140.14	125.391485	10.5241	20.5074

Tuning Performance Classical PSO

Classical Particle Swarm Optimization (PSO) has been effectively applied for tuning natural frequencies and corresponding vibration modes in finite element model updating (FEMU). PSO was employed to minimize a cost function combining the relative errors in natural frequencies and the Modal Assurance Criterion (MAC). The objective was to align a FEM with target modal data comprising three modes. The PSO algorithm used a swarm size of 100, with 100 maximum iterations, an inertia weight of 0.7, and where the cognitive coefficient and the social coefficient are each fixed at 1.5. Initially, the frequency errors were relatively high, averaging **6.8%,** with the corresponding mode shape discrepancies showing **MAC errors** ((1-"MAC")×100)ranging between **7.0568%** and **20.5074%**. After tuning using the **Classical PSO**, the average frequency error was reduced to just 0.33%, with individual mode errors of **0.4020%**, **0.0871%**, and **0.4917%** for the first, second, and third bending modes, respectively. The optimization converged in approximately **85 iterations**, effectively capturing the physical behavior of the structure. These results highlight the **robustness and efficiency** of the Classical PSO algorithm in finite element model updating, providing a straightforward yet powerful approach to inverse problems in structural dynamics. The tuning performance of Classical PSO is presented in **Table 2**, and the comparative mode shape results are illustrated in **Figure 5**.

Table 2: Result from Classical PSO

Mode	Target Frequency	Frequency from PSO	Error	MAC Error ((1-MAC)×100)%
1st Bending Mode	31.73	31.6023979	0.4020	8.8932
2nd Bending Mode	73.96	73.8954660	0.0871	4.7436
3rd Bending Mode	140.14	139.451849	0.4917	18.0067

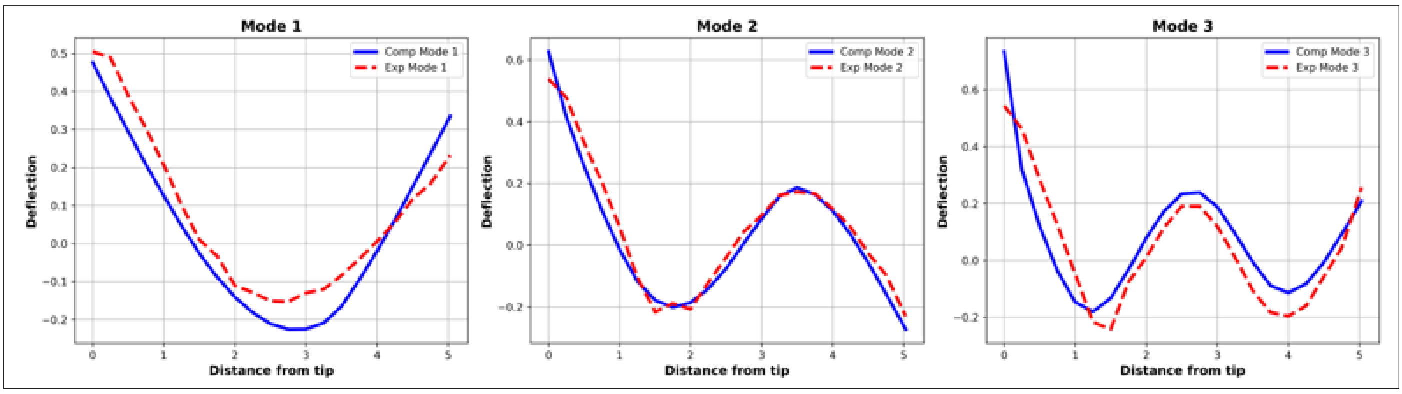

Figure 5: Mode shape comparison of result from QPSO

Tuning Performance qPSO

In this study, QPSO was applied to a finite element model updating problem involving three target modes, with the objective of minimizing the relative frequency error and maximizing the Modal Assurance Criterion (MAC). The algorithm used a swarm of 100 particles over 100 iterations, with β linearly decreasing from 1.0 to 0.5. Prior to tuning, the initial finite element model had average frequency errors of 6.8%. After **QPSO-based updating**, the frequency errors were reduced to **0.1328%**, **0.7226%**, and **0.0890%** for the first three bending modes, giving an average frequency error of only **0.31%**. The algorithm consistently converged in fewer than **60 iterations**, demonstrating faster convergence and higher accuracy compared to the classical PSO. The corresponding **MAC errors** were **8.8163**, **4.8181**, and **18.1716**, showing strong agreement between the experimental and analytical mode shapes. The tuning performance of QPSO is presented in **Table 3**, and the comparative mode shape results are illustrated in **Figure 6**.

Table 3: Result from QPSO

Mode	Experimental Frequency	Frequency from QPSO	Error	MAC Error ((1-MAC)×100)
1st Bending Mode	31.73	31.7718439	0.1328	8.8163
2nd Bending Mode	73.96	74.4937616	0.7226	4.8181
3rd Bending Mode	140.14	140.015052	0.0890	18.1716

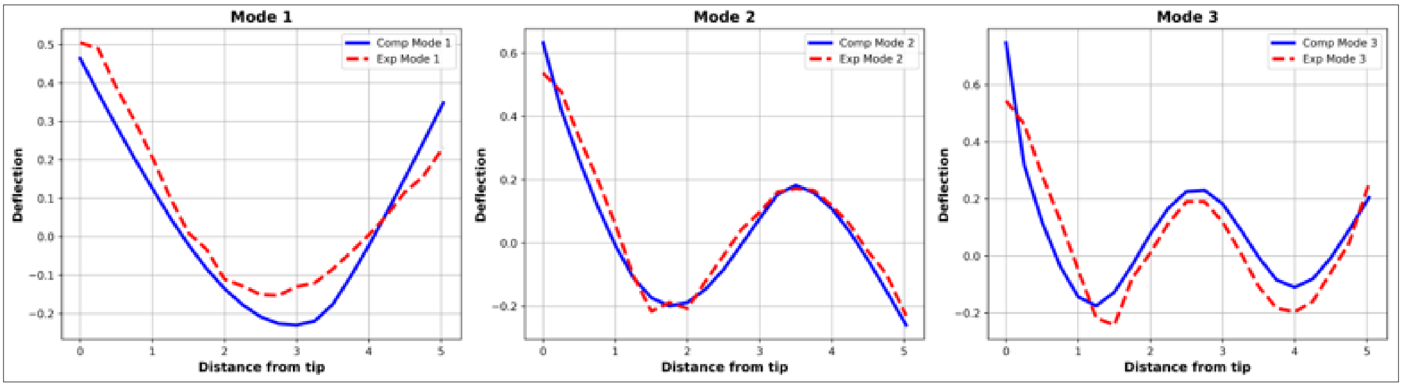

Figure 6: Mode shape comparison for QPSO

Tuning Performance APSO

Adaptive Particle Swarm Optimization (APSO) enhances the classical PSO framework by dynamically adjusting inertia weight and learning coefficients during the optimization process, enabling better control over the exploration–exploitation trade-off. In this study, APSO was applied to update a FEM based on three target modal frequencies and mode shapes. The inertia weight $\omega(t)$ was linearly decreased from 1 to 0.4, while the cognitive coefficient $c_1(t)$ decreased from 2.6 to 0.5 and the social coefficient $c_2(t)$ increased from 0.5 to 2.6 throughout 100 iterations. Using a swarm of 100 particles, after updating, the frequency errors for the first three bending modes were **0.1036%**, **0.3678%**, and **0.1898%**, respectively, with an average of approximately **0.22%**. The corresponding **MAC errors** were **8.7215, 4.9753**, and **18.0842**, indicating strong correlation between the analytical and target mode shapes. The detailed results of the APSO-based updating are summarized in **Table 4**, while the comparative mode shape results are illustrated in **Figure 7**.

Table 4: Result from APSO

Mode	Experimental Frequency	Frequency from APSO	Error	MAC Error ((1-MAC)×100)
1st Bending Mode	31.73	31.6971437	0.1036	8.7215
2nd Bending Mode	73.96	74.2319903	0.3678	4.9753
3rd Bending Mode	140.14	139.874717	0.1898	18.0842

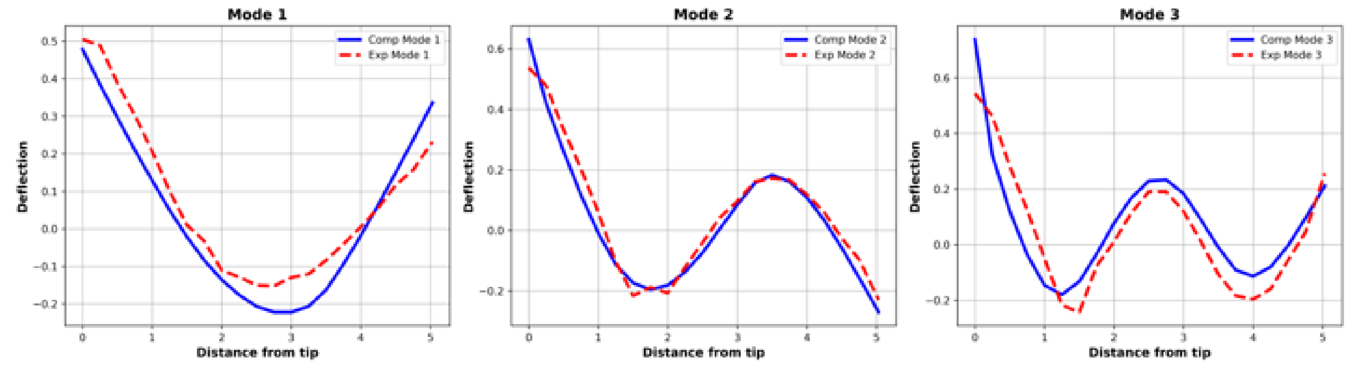

Figure 7: Mode shape comparison APSO

Comparative Analysis

A comparative analysis of Classical PSO, QPSO, and APSO highlights clear differences in tuning performance, convergence behavior, and solution accuracy when applied to finite element model updating (FEMU) using modal frequency and mode shape data. The Classical PSO, though simple and reliable, achieved an average frequency error of 0.332% for the first three modes and required approximately 85 iterations to converge. The QPSO algorithm improved the results through a more global exploration strategy, reducing the average frequency error to 0.314%, with convergence typically reached within 60 iterations. The APSO algorithm demonstrated the best overall performance, using adaptive parameter control to further reduce the average frequency error to 0.22%, with convergence achieved in about 80 iterations.

Beyond improved accuracy, APSO also exhibited greater stability across multiple runs and produced more physically consistent parameter updates, making it especially suitable for damage detection and high-fidelity model calibration. While all three algorithms effectively minimized the objective function, the adaptive and quantum-based mechanisms in APSO and QPSO provided superior robustness, convergence speed, and precision compared to the classical approach.

Conclusions

This work examines the application of Particle Swarm Optimization (PSO) algorithms to refine modal frequencies and mode shapes within the framework of finite element model updating (FEMU). Three variants of PSO—Classical PSO, Quantum-behaved PSO (QPSO), and Adaptive PSO (APSO)—were implemented and evaluated for their performance to minimize the differences between computational models and reference modal data. The objective was to reduce relative frequency errors and increase the Modal Assurance Criterion (MAC), thereby improving the predictive capability of structural models in dynamic analysis.

The comparative analysis confirmed that while all three algorithms are capable of tuning finite element models with high accuracy, APSO consistently outperformed the others in terms of precision, and stability. And QPSO outperformed in terms of convergence speed. These findings emphasize the importance of adaptive parameter control in metaheuristic optimization for structural dynamics applications.

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